8-9

Initial cost = $100,000

Additional cost at the end of Year 1 = $150,000

Benefit at end of Year 1 = 0

Benefit at the end of Years 2–10 = $20,000 per year

Time = 10 years

Interest rate = 7%

PW of benefits = Benefit [*P/A*, 7%, 9 (since benefit is from 2 to 10 years)] (*P/F*, 7%, 1) (since it is for only 1 year)]

= $20,000 (6.515) (0.9346)

= $20,000 (6.089)

= $121,778

PW of costs = Initial cost + Additional cost after 1 year (*P/F*, 7%, 1)

= $100,000 + $150,000 (0.93461)

= $100,000 + $140,191.5

= $240,191

**B/C ratio** = PW of benefits/PW of costs

= $121,778.38/$240,191

= **0.507**

8-11

The problem can be solved using calculus, though we can also use a spreadsheet to solve it graphically. The project’s “optimal size” is the size at which the B/C ratio is at a maximum.

The first step is to calculate benefits for a range of costs. We can re-arrange the given equation to get:

PW(benefits) = [-44+22 × PW(costs)]0.5

Taking a range of costs, we use this to plot Figure 8.11(i):



Figure 8.11(i)

Using this set of values of the present worths of benefits and costs, we now plot a second graph, of PW(costs) versus B/C ratio:

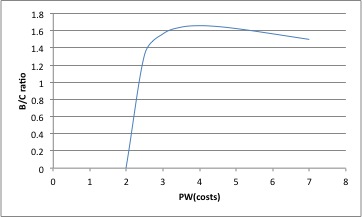


Figure 8.11 (ii)

From this we can see that the B/C ratio reaches a maximum of 1.66 when the present worth of costs is $4,000,000.

8-16

This problem requires some thought on how to structure the analysis. This is a situation of providing the necessary capacity when it is needed—in other words Fixed Output. Computing the cost is easy, but what is the benefit?

We cannot compute the B/C ratio for either alternative, but we can compute the incremental B/C ratio on the difference between alternatives.

We start with the assumption that we’re going to build the full-capacity tunnel, and consider the B/C ratio of upgrading to the half-capacity tunnel. This upgrade has the benefit of saving us $200,000 now, $4,000 in lining repair in 10 and 20 years’ time, but it will then cost us $400,000 for the second tunnel, and an additional $12,000 for lining repairs in 30 and 40 years’ time

So ΔB/ΔC (all in $000)

= (200+4((*P/F,* 0.05,10)+ ((*P/F,* 0.05,20)))/(400(*P/F*, 0.05,20)+12((*P/F*, 0.05,30)+ *P/F,* 0.05,40)))

= (200+4((0.6139) + (0.3768))) / (400(0.3768) + 12((0.2324) + (0.1420)))

= 204.0/155.2 = **1.314**

**So the better choice is to build the half-capacity tunnel now.**

8-19

Upfront investment required = $275,000

The benefit available in the first year is $10,000; thereafter, it increases by $2,000 each year.

The upfront investment will break even only if PW of costs = PW of benefits or,

PW of benefits – PW of costs = 0

Let the end-of-year benefit in the year we start the project = *X*

*X* (*P/A*, 6%, 15) + $2,000 (*P/G*, 6%, 15) – $275,000 = 0

9.712 *X* + $2,000 (57.555) = $275,000

*X* = [$275,000 – $2,000 (57.555)]/9.712

= ($275,000 – $115,110)/9.712

= 159,890/9.712

= $16,463

It is clear that the investments should be made only when the benefit at the end of the first year is greater than $16,643, which happens only in 2020. Hence, no construction should be done before 2020.

Now we must decide whether the construction should take place in 2020 or 2021. To answer this, we will compute the NPW for both the years:

**NPW in 2020** = $18,000 (*P/A*, 6%, 15) + $2,000 (*P/G*, 6%, 15) – $275,000

= $18,000 (9.712) + $2,000 (57.555) – $275,000

= $174,816 + $115,110 – $275,000

= **$14,926**

**NPW in 2021** = [$20,000 (*P/A*, 6%, 15) + $2,000 (*P/G*, 6%, 15) – $275,000] (*P/F*, 6%, 1)

= [$20,000 (9.712) + $2,000 (57.555) – $275,000] (0.9434)

= ($194,240 + $115,110 – $275,000) (0.9434)

=$34,350 (0.9434)

= **$32,406**

Since the NPW is greater in 2021, **the construction should take place in 2021**.

8-27

Payback Period = Cost/Annual Benefit = $3,800/4($400) = 2.4 years



$3,800 = $400 (*P*/*A*, *i*%, 4) + $400 (*P*/*A*, *i*%, 4) (*P*/*F*, *i*%, 12) + $400 (*P*/*A*, *i*%, 4) (*P*/*F*, *i*%, 24) + $400 (*P*/*A*, *i*%, 4) (*P*/*F*, *i*%, 36) + $400 (*P*/*A*, *i*%, 4) (*P*/*F*, *i*%, 48)

$3,800 = $400 (*P*/*A*, *i*%, 4) [1 + (*P*/*F*, *i*%, 12) + (*P*/*F*, *i*%, 24) + (*P*/*F*, *i*%, 36) + (*P*/*F*, *i*%, 48)]

Try *i* = 3%

P(3%) = $400 (3.717) [1 + 0.7014 + 0.4919 + 0.3450 + 0.2420] = $1,486.80 [2.7803] = $4,134 *i* too low

Try *i* = 4%

P(4%) = $400 (3.630) [1 + 0.6246 + 0.3901 + 0.2437 + 0.1522]

= $1,452 [2.4106]

= $3,500 *i* too high

Try *i* = 3.5%

P(3.5%) = $400 (3.673) [1 + 0.6618 + 0.4380 + 0.2898 + 0.1918]

= $1,469.20 [2.5814]

= $3,798

So *i* = 3.5% per month

Nominal Rate of Return = 12 (3.5%) = 42%

8-29

**Lease:** A = $5,000/yr

**Purchase:**

$7,000 A = $3,500

S = $500

(a) Payback Period

Cost = $7,000

Benefit = $1,500/yr + $500 at any time

Payback = ($7,000 – $500)/$1,500 = 4.3 years

(b) Benefit–Cost Ratio

B/C = EUAB/EUAC

= [$1,500 + $500 (A/F, 10%, 6)]/[$7,000 (*A*/*P*, 10%, 6)]

= [$1,500 + $500 (0.1296)]/[$7,000 (0.2296)]

= 0.97